

(3) Pointwise Convergence: - Let  $\langle f_n \rangle$  be a sequence of measurable functions defined over a measurable set  $E$ . If there exists a measurable function  $f$  over  $E$ .

$$\text{s.t. } \lim_{n \rightarrow \infty} f_n(x) = f(x) \quad \forall x \in E$$

then we say that the sequence  $\langle f_n \rangle$  converges pointwise to  $f$  on  $E$ .

(4) Convergence Almost Everywhere: - Let  $\langle f_n \rangle$  be a sequence of measurable functions defined over a measurable set  $E$ . If  $\exists$  a measurable function  $f$  over  $E$  and a set  $A$ .

$$\text{s.t. } (i) \quad m(A) = 0$$

$$(ii) \quad \lim_{n \rightarrow \infty} f_n(x) = f(x) \quad \forall x \in E - A$$

(i.e.,  $\langle f_n \rangle$  converges pointwise to  $f$  on  $E - A$ )  
then we say that the sequence  $\langle f_n \rangle$  converges to  $f$  almost everywhere on  $E$ .

### Theorems on Convergence of Sequences of Measurable Functions:

(1) Convergence in Mean: - A sequence  $\langle f_n \rangle$  of Lebesgue integrable functions is said to converge in mean to a function  $f$ .

$$\text{if } \lim_{n \rightarrow \infty} \int_E |f_n - f| dx = 0$$

(2) Convergence in measure: - Let  $\langle f_n \rangle$  be a sequence of measurable functions defined over a measurable set  $E$ . Let  $f$  be a measurable function defined over  $E$ .

$$\text{s.t. } (i) \quad f(x) < \infty \quad \text{a.e. on the set } E$$

$$(ii) \quad \lim_{n \rightarrow \infty} m[E(|f_n - f| \geq \epsilon)] = 0 \quad \forall \epsilon > 0$$

Then the sequence  $\langle f_n \rangle$  is said to converge in measure to the function  $f$ .